This article first appeared in the July 1944 issue of "Electronics".

Tube Filament and Heater Characteristics

By CECIL E. HALLER

Mathematical analysis of volt-ampere characteristics of various filament metals. Resulting equations are plotted as a reference chart that gives filament current, temperature, and wattage for any electron tube at various operating voltages with good accuracy

FREQUENTLY in the design and application of electron tubes, it is necessary to predict the value of filament current and possibly also the filament temperature when the applied filament voltage deviates from the normal or rated value. Such an instance occurs when two or more filaments having different volt-ampere characteristics are operated in series and it is required to predict the voltage variation across each individual filament with respect to the supply voltage variation. The solution of this problem requires a knowledge of the volt-ampere characteristic of each filament. The voltage across each filament can then be determined for any arbitrary assumed current value. If appropriate current values are chosen, a curve of each individual tube voltage vs. the supply voltage can be constructed.

A knowledge of the individual volt-ampere characteristics over the required range may not be readily available unless it has previously been experimentally determined. It is the purpose here to indicate a method of constructing the volt-ampere characteristic if the current is known for at least one operating voltage. The knowledge of this point enables the current to be predicted at a new operating voltage with good accuracy provided that the change in voltage is within approximately ±25 per cent of the known voltage. The same analysis will also permit the determination of the value of watts and temperature in terms of the known operating condition.

Basic Filament Equations

In order to illustrate the method of transposing the operating condition of a filament, the two basic equations involved in the design of filaments and heaters for electron tubes will be considered.

The first equation is

$$W = K_1 T^{n_w} \tag{1}$$

where W = power radiated, watts
K_1 = constant of proportionality, which includes the area of the emitter
T = temperature, °K
n_w = an exponent that is reasonably constant for a given metal over a limited range of temperature

The second equation is

$$W = \frac{E^2}{R} = I^2 R \tag{2}$$

where E = applied filament voltage
R = resistance of the filament, which is in general a function of temperature
I = filament current

In order to solve Eqs. (1) and (2), let

$$R = K_2 T^{n_r} \tag{3}$$

where K_2 = a constant of proportionality
n_r = an exponent which may be regarded constant over a limited temperature range

Solutions of Eqs. (1), (2), and (3) for W, I, and T in terms of E yield[1]

$$W = K_1 (K_1 K_2)^{\frac{2n_w}{n_w + n_r}} (E)^{\frac{2n_w}{n_w + n_r}} \tag{4}$$

$$I = \left(\frac{K_1}{K_2}\right)^{\frac{n_w - n_r}{2}} \left(\frac{1}{K_1 K_2}\right)^{\frac{2}{n_w + n_r}} (E)^{\frac{n_m - n_r}{n_w + n_r}} \tag{5}$$

$$T = \left(\frac{1}{K_1 K_2}\right)^{\frac{2}{n_w + n_r}} (E)^{\frac{2}{n_w + n_r}} \tag{6}$$

Thus, when using W_o, I_o, T_o, E_o and W_x, I_x, T_x, E_x as the known and unknown conditions, respectively, Eqs. (4), (5), and (6) may be written

$$\frac{W_x}{W_0} = \left(\frac{E_x}{E_0}\right)^{\frac{2n_w}{n_w + n_r}} \tag{7}$$

$$\frac{I_x}{I_0} + \left(\frac{E_x}{E_0}\right)^{\frac{n_w - n_r}{n_w + n_r}} \tag{8}$$

$$\frac{T_x}{T_0} = \left(\frac{E_x}{E_0}\right)^{\frac{2}{n_w + n_r}} \tag{9}$$

Use of Average Values for Exponents

Values for n_w have been determined for some of the more common metals, and are given in Table I. The exponent n_w includes the change of total emissivity with temperature. Since the total emissivity of metals increases with tempera-

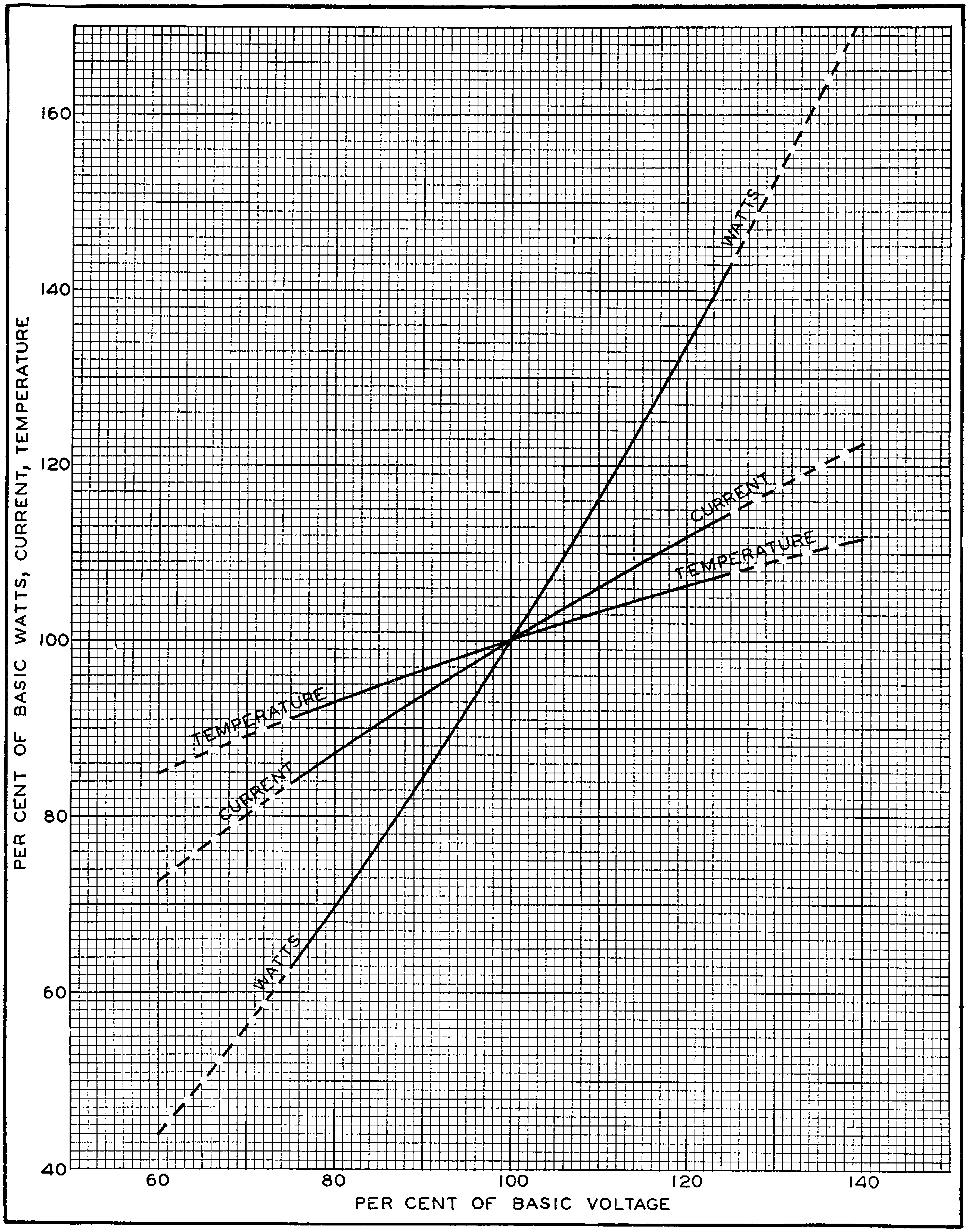

Fig. 1.—Chart giving wattage, current, and temperature of a filament or heater at operating voltages up to 25 per cent above or below basic voltage with sufficient accuracy for most engineering purposes. Accuracy drops in dotted regions.

Table I.—Values of Exponents for Four Filament Metals

Material	Temp., °K	n_w	n_r	$\frac{2n_w}{n_w + n_r}$	$\frac{n_w - n_r}{n_w + n_r}$	$\frac{2}{n_w + n_r}$
Tungsten[1]...............	1000	5.65	1.20	1.65	0.650	0.292
Tungsten[1]...............	2000	4.93	1.19	1.61	0.612	0.327
Tungsten[1]...............	2500	4.66	1.20	1.59	0.590	0.341
Molybdenum[3]...........	1000	5.32	1.15	1.64	0.645	0.309
Molybdenum[3]...........	2000	4.99	1.15	1.63	0.625	0.326
Tantalum[3]..............	1600	4.80	0.785	1.72	0.720	0.358
Tantalum[3]..............	2800	4.80	0.785	1.72	0.720	0.358
Nickel....................	1000	4.65†	0.62†	1.76	0.764	0.379

* Values by different investigators range from 4.65 to 5.29.
† Estimated from experimental data on nickel.

ture,[2] the Stefan-Boltzmann law of radiation requires that n_w be greater than 4. For materials having an emissivity independent of temperature, the value will be 4. The value of n_r for some metals at different temperatures is also given in Table I. These limited data indicate n_r ranges from 0.6 to 1.2.

If W_1 and W_2, the watts radiated at temperatures T_1 and T_2, are known, then

$$n_w = \frac{\log (W_1/W_2)}{\log (T_1/T_2)}$$

In a like manner, if R_1 and R_2 are the resistances at temperatures T_1 and T_2, then

$$n_r = \frac{\log (R_1/R_2)}{\log (T_1/T_2)}$$

These permit an experimental check of the values of n_w and n_r if two sets of operating conditions are known.

Table I also gives the value of $(n_w - n_r)/(n_w + n_r)$. It will be noted that it ranges from 0.59 to 0.76. It is now of interest to see what error results in using an average value of this exponent in Eq. (8). A voltage ratio E_x/E_0 of 1.25 will be taken as the maximum voltage for which Eq. (8) is to be used. If the exponents 0.59 and then 0.76 are used, the respective values of current ratio I_x/I_0 are 1.140 and 1.185, or a deviation of only ±1.9 per cent from the mean value.

Plotting the Chart

It is evident therefore that an average exponent can be chosen which applies to all the metals in Table I and in general

$$\left(\frac{I_x}{I_0}\right) = \left(\frac{E_x}{E_0}\right)^{0.61}, \qquad 0.75 < \left(\frac{E_x}{E_0}\right) < 1.25 \tag{10}$$

is true to sufficient accuracy for most engineering purposes. This equation has been drawn as the current curve for the chart in Fig. 1.

It can further be shown that the accuracy involved in assuming $2n_w/(n_w + n_r) = 1.61$ and $2/(n_w + n_r) = 0.327$ is even greater than for the case just discussed. Then

$$\frac{W_x}{W_0} = \left(\frac{E_x}{E_0}\right)^{1.61} \tag{11}$$

$$\frac{T_x}{T_0} = \left(\frac{E_x}{E_0}\right)^{0.327} \tag{12}$$

These equations are plotted in Fig. 1.

In order to check Eq. (8) and Fig. 1, data were taken for a wide variety of electron tubes. It will be noted from Fig. 1 and Table II that the calculated values of current deviate, in general, by less than ±4 per cent from the measured values. We can conclude from this that Fig. 1 is generally applicable to all types of electron tubes for the specified range of $(E_x/E_0)^5$. Some samples of the use of this curve will now be given.

Example 1.—The type RCA-826 has a thoriated-tungsten filament rated at 7.5 volts and 4 amp. What will be the filament current at 5.62 volts (75 per cent of rated voltage)? Tracing up from 75 on the horizontal scale in Fig. 1 to the current curve, and then across, gives 83.8 per cent. The new filament curve is then $0.838 \times 4 = 3.35$ amp. By actual measurement, the current was found to be 3.34 amp. While no temperature measurements were made at this voltage, one would expect the temperature to decrease to 91 per cent of its rated value in degrees Kelvin.

2. An oxide-coated cathode has a temperature of 1000°K when the heater is operated at 5 volts. What voltage will be required to increase the temperature to 1060°K? From Fig. 1, when $T_x/T_0 = 106$, one finds $E_x/E_0 = 1.19$ or $E_x = 5 \times 1.19 = 5.95$ volts. The heater voltage was found to be 6.0 volts when determined experimentally. If the voltage had been increased to 7 volts, then $E_x/E_0 = 1.4$ and $T_x/T_0 = 1.118$ or $T_x =$ 1118°K. Actual measurements indicated the temperature to be 1135°K.

Two Filaments in Series

In order to examine the operation of filaments or heaters in series, let two tubes T_1 and T_2, having the same nominal voltage rating, be connected in series to a power supply E_s (see Fig. 2). In general, the filaments of tubes T_1 and T_2 may have different volt-ampere characteristics. Such differences may be due to the individual variation of filament current when read at a specified or rated voltage. These variations of filament currents, which are expected and normal, result from the necessary manufacturing tolerances on both materials and processes.

The usual filament-current tolerance on receiving and the smaller transmitting tubes, whose filaments or heaters might be operated in series, is generally of the order of 5 to 10 per cent. In order to ensure satisfactory operation initially and throughout the expected life of the tube, the tube manufacturer usually specifies the percentage the applied filament voltage may be allowed to deviate from normal. This voltage deviation is usually of the order of ±5 per cent for the thoriated-tungsten type emitters and ±10 per cent for the oxide-coated filament or heater-cathode types. It becomes apparent that for series operation of the filament or heaters the supply-voltage variation needs careful consideration in order to ensure that the individual filament-voltage tolerances are not exceeded on either T_1 or T_2.

If T_4 and T_2 are both high or both low filament-current tubes, no problem exists since the volt-ampere characteristics are substantially identical (*i.e.*, they have the same currents for the same applied voltage). Only the combination needs to be considered, where one has the higher limit value and the other has the lower limit value of filament current.

Determining Permissible Filament Voltage Variations

As a specific problem, assume the filament-current tolerance to be ±5 per cent and let it be required to determine the permissible variation of supply voltage without exceeding an individual tube voltage range of ±10 per cent. This problem frequently arises in the design of mobile transmitters when the filament or heaters are operated in series.

The bogie or normal volt-ampere characteristic *BKE* of Fig. 2 may then be constructed from Eq. (10). For T_1, the 5 per cent high filament-current tube, the equation of the volt-ampere characteristic is

$$\frac{I_x}{I_0} = 1.05 \left(\frac{E_x}{E_0}\right)^{0.61} \tag{13}$$

Curve *AJD* can be constructed from this equation.

Similarly for T_2, the low filament-current tube, the equation is

$$\frac{I_x}{I_0} = 0.95 \left(\frac{E_x}{E_0}\right)^{0.61} \tag{14}$$

which is represented by the curve *CLF* in Fig. 2.

An arbitrary current may then be assumed in order to determine the individual voltage across tubes T_1 and T_2. The supply voltage is represented by the sum of the voltages across T_1 and T_2. If T_1 (the high filament-current tube) has the minimum permissible voltage 0.9 E_0 represented by point *A*, tube T_2 (the low filament-current tube) will have a voltage represented by point *O*. The sum of the two voltages at points *A* and *O* represents the minimum permissible supply voltage consistent with the previous imposed condition of ±5 per cent filament-current tolerance

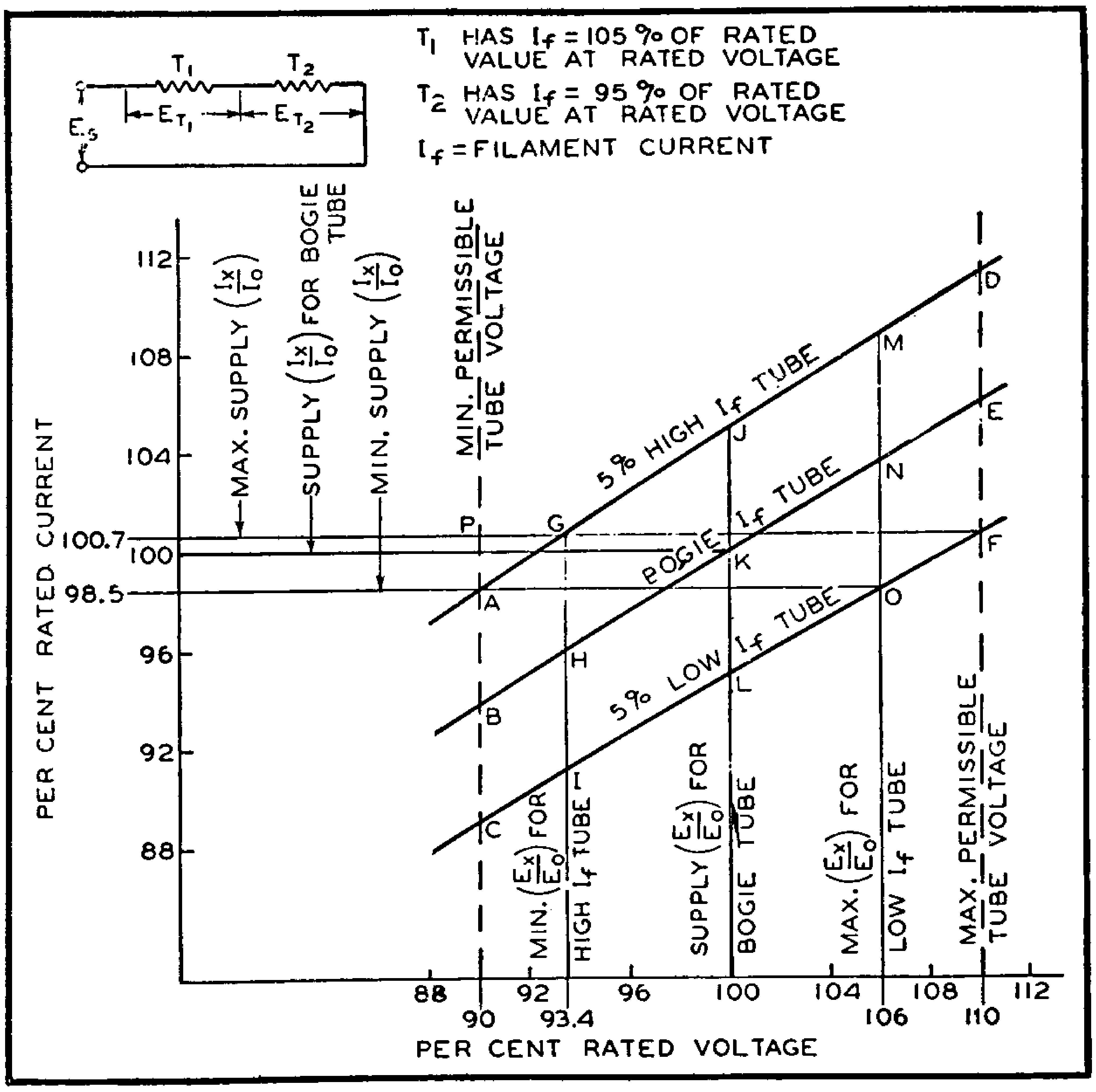

Fig. 2.—Volt-ampere characteristics for series operation of filaments or heaters having different characteristics.

Table II.—Measured and Calculated Data on Typical Tube Samples

Type number	Filament or heater	Type of emitter	Rate voltage		Reduced voltage				Increased voltage			
			E_{x0}	I_{x0} observed	E_{x1}	I_{x1} observed	I_{x1} calculated from Fig. 1	% difference	E_{x2}	I_{x2} observed	I_{x2} calculated from Fig. 1	% difference
203A	Filament	Th-W	10.0	3.28	7.5	2.72	2.74	+0.7	12.5	3.70	3.76	+1.6
801A	Filament	Th-W	7.5	1.22	5.62	1.05	1.02	−0.3	9.38	1.42	1.40	−1.4
807	Cathode	Oxide	6.3	0.87	4.72	0.74	0.73	−1.4	7.88	0.99	1.00	+1.0
813	Filament	Th-W	10.0	5.00	7.50	4.12	4.18	+1.5	12.5	5.80	5.72	−1.4
815	Cathode	Oxide	6.3	1.65	4.72	1.39	1.37	−1.4	7.88	1.88	1.89	+0.5
826	Filament	Th-W	7.5	4.00	5.62	3.35	3.34	−0.3	9.38	4.55	4.58	+0.7
833A	Filament	Th-W	10.0	10.10	7.5	8.50	8.44	−0.7	12.50	11.60	11.56	−0.3
836	Cathode	Oxide	2.5	5.05	1.88	4.22	4.25	+0.7	3.13	5.75	5.78	+0.5
861	Filament	Th-W	11.0	10.05	8.25	8.30	8.40	+0.1	13.75	11.50	11.50	0
866	Filament	Oxide	2.5	5.00	1.87	4.03	4.18	+3.7	3.13	5.80	5.72	−1.4
913	Cathode	Oxide	6.3	0.600	4.72	0.504	0.502	−0.4	7.88	0.685	0.687	+0.3
1616	Filament	Oxide	2.5	4.90	1.87	3.97	4.10	+3.3	3.13	5.73	5.61	−2.1
1624	Filament	Oxide	2.5	1.83	1.87	1.47	1.53	+4.1	3.13	2.12	2.09	−1.4
2050	Cathode	Oxide	6.3	0.575	4.72	0.489	0.481	−1.6	7.88	0.657	0.657	0
2051	Cathode	Oxide	6.3	0.605	4.72	0.515	0.506	−1.8	7.88	0.688	0.693	+0.6
8025	Filament	Th-W	6.3	1.94	4.72	1.65	1.61	−2.4	7.88	2.19	2.22	+1.4
9001	Cathode	Oxide	6.3	0.157	4.72	0.135	1.31	−3.0	7.88	0.177	0.179	+1.1
1T4	Filament	Oxide	1.4	0.0525	1.05	0.043	0.044	+2.3	1.75	0.0613	0.0600	−2.2
6SK7	Cathode	Oxide	6.3	0.310	4.72	2.61	2.59	−0.8	7.88	0.352	0.355	+0.9
6SS7	Cathode	Oxide	6.3	0.150	4.72	0.129	1.25	−3.1	7.88	0.170	0.172	+1.2
12A6	Cathode	Oxide	12.6	0.156	9.45	0.134	1.30	−3.0	15.75	0.177	0.178	+0.6
2AP1	Cathode	Oxide	6.3	0.595	4.72	0.508	0.498	−2.0	7.88	0.668	0.680	+1.7

and ±10 per cent individual voltage tolerance.

In a similar manner the maximum voltage 1.1 E_0 that can be applied to T_2, the low filament-current tube, is represented by point F, and the voltage on T_1, the high filament-current tube, by point G. The sum of the voltages at points G and F gives the maximum permissible supply voltage. The nominal supply voltage is twice the nominal tube voltage or $2E_0$ since it was assumed that the nominal voltage ratings of T_1 and T_2 were identical. The maximum permissible percentage of supply voltage deviation may then be calculated directly from the graphical analysis. This solution indicates supply voltage tolerance of +1.7 per cent and −2.0 per cent.

It is interesting to note that the distance A to P represents the maximum permissible range of supply current. This suggests that if the supply voltage cannot be maintained within the required limits, a series ballast tube, whose current is maintained within the range A to P, might be used to permit a larger variation in the supply voltage. The use of the ballast tube would of course require an increased supply voltage in order to supply the required voltage drop of the ballast tube.

An alternative solution to permit wider supply-voltage tolerances consists in shunting the low filament-current tube with a resistor. This resistor is adjusted until both tubes have substantially the same filament voltage. This method is essentially one of shifting the operating point on the volt-ampere characteristic of the low filament-current tube and resistor until at normal supply voltage it coincides with that of the high filament-current tube. This method does not, however, make the volt-ampere characteristics identical and, therefore, never can permit a percentage supply-voltage change equal to the permitted percentage of individual filament voltages. In order to simplify adjustments, adjustable resistors are frequently used across both filaments.

Generalized Solution for Tubes in Series

The method of the solution of two tubes in series may be generalized for N_H tubes having high filament currents in series with N_L tubes having low filament currents, as shown in Fig. 3.

Let m = percentage filament-current tolerance
p = permissible percentage tolerance of individual applied voltage

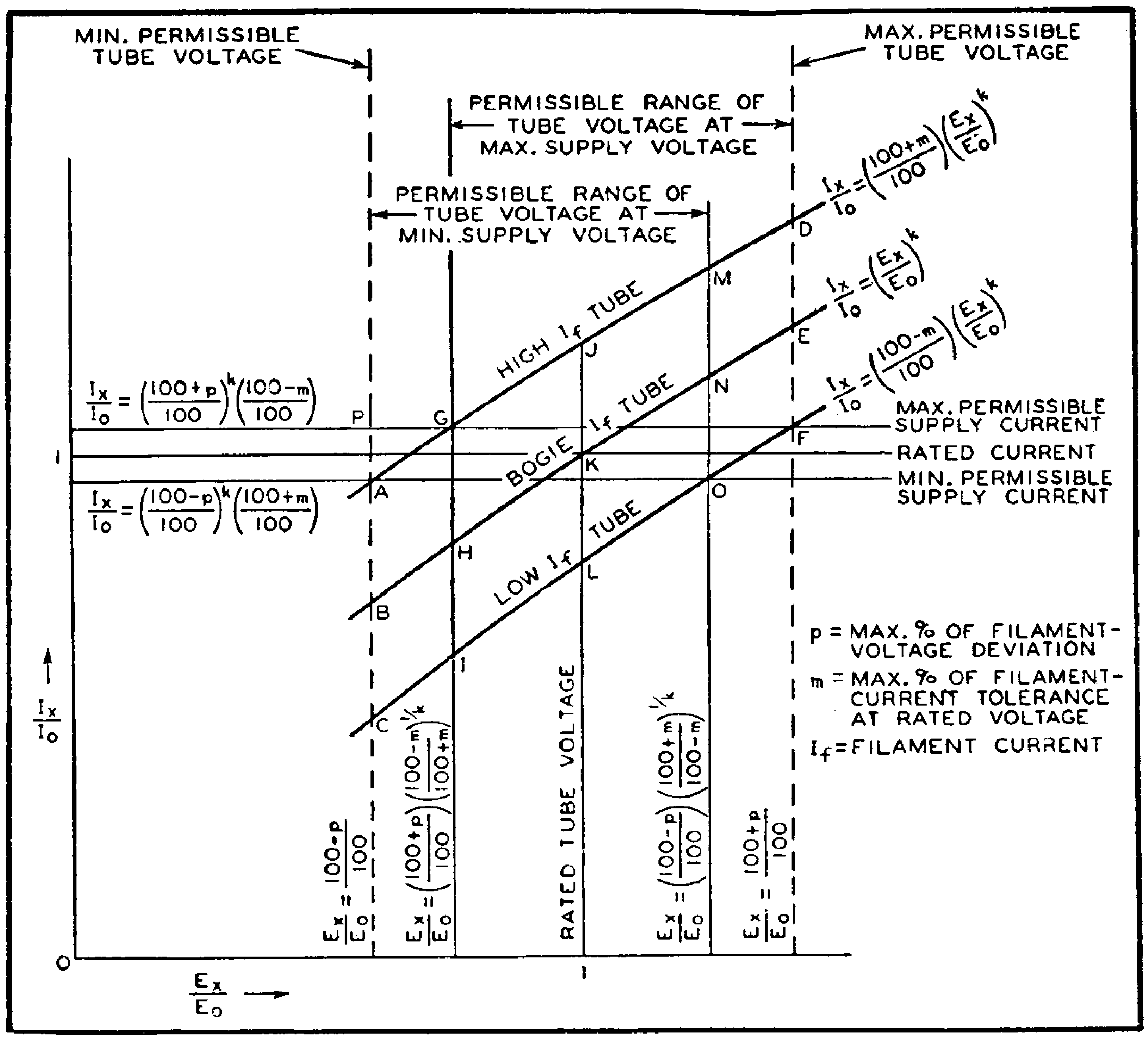

Fig. 3.—Generalized solution for any number of tubes in series, some having high and some low filament current.

The volt-ampere characteristic of a limit tube is expressed by

$$\left(\frac{I_x}{I_0}\right) = \left(\frac{100+m}{100}\right)\left(\frac{E_x}{E_0}\right)^{0.61} \quad (15)$$

But since E_x/E_0 for the limiting condition of maximum voltage on a low filament-current tube is equal to $(100+p)/100$, Eq. (15) may be rewritten for a low filament-current tube as

$$\left(\frac{I_x}{I_0}\right)_{\max} = \left(\frac{100+p}{100}\right)^{0.61}\left(\frac{100-m}{100}\right) \quad (16)$$

where I_x/I_0 is the maximum permissible supply current when at least one each of limit values of high and low filament-current tubes are operated in series. Similarly, the minimum permissible current is given by

$$\left(\frac{I_x}{I_0}\right)_{\min} = \left(\frac{100-p}{100}\right)^{0.61}\left(\frac{100+m}{100}\right) \quad (17)$$

Equations (14) and (15) form the basis for calculating the current requirements imposed on a series ballast tube should one be used.

The values of current given in Eqs. (16) and (17), when substituted in the appropriate equations for the volt-ampere characteristics, give the voltage at points G and O, respectively, as

$$\left(\frac{E_x}{E_0}\right) = \left(\frac{100+p}{100}\right)\left(\frac{100-m}{100+m}\right)^{1.64} \quad (18)$$

and

$$\left(\frac{E_x}{E_0}\right) = \left(\frac{100-p}{100}\right)\left(\frac{100+m}{100-m}\right)^{1.64} \quad (19)$$

The value of (E_x/E_0) given in Eq. (18) represents the voltage across a high filament-current tube at the maximum permissible supply voltage, while the value given in Eq. (19) represents the voltage across a low filament-current tube at the minimum permissible supply voltage. The sum of N_H voltages given in Eq. (18) $+N_L$ voltages of the value $(100+p)/100$ gives the maximum supply voltage or

$$\left(\frac{E_s}{E_0}\right)_{\max} = N_H\left[\left(\frac{100+p}{100}\right)\left(\frac{100-m}{100+m}\right)^{1.64}\right] + N_L\left(\frac{100+p}{100}\right) \quad (20)$$

In a like manner, the minimum supply voltage is

$$\left(\frac{E_s}{E_0}\right)_{\min} = N_L\left[\left(\frac{100-p}{100}\right)\left(\frac{100+m}{100-m}\right)^{1.64}\right] + N_H\left(\frac{100-p}{100}\right) \quad (21)$$

The percentage of supply voltage tolerance becomes

$$\% \ E_s \text{ above normal} = \left\{\frac{N_H\left[\left(\frac{100+p}{100}\right)\left(\frac{100-m}{100+m}\right)^{1.64}\right] + N_L\left(\frac{100+p}{100}\right)}{N_H + N_L} - 1\right\} 100 \quad (22)$$

$$\% \ E_s \text{ below normal} = \left\{ \frac{N_L \left[\left(\frac{100 - p}{100} \right) \left(\frac{100 + m}{100 - m} \right)^{1.64} \right] + N_H \left(\frac{100 - p}{100} \right)}{N_H + N_L} - 1 \right\} 100 \quad (23)$$

Equations (22) and (23) were derived on the premise that at least one tube of the group had a filament current $(100 + m)/100$ times rated value and at least one other had $(100 - m)/100$ times rated value. This premise imposes the condition that both N_H and N_L must be different from zero in Eqs. (22) and (23). If either N_H or N_L is zero, the solution is simple as all possess the same volt-ampere characteristic. The permissible supply-voltage percentage deviation is p.

Application to Universal Receivers

It is common practice in the design of a-c/d-c sets in which the filaments or heaters are connected in series to use tubes that have different values of rated filament voltage. The analysis of this condition can be reduced to an expression similar to Eqs. (22) and (23) except that N_H and N_L must include the equivalent number of respective tubes in terms of the lowest nominal voltage tube. For example, a 35-volt tube may be represented by 5.55 tubes of 6.3 volts nominal rating.

Substitution in Eqs. (22) and (23) of the conditions of the previously discussed case of two tubes ($N_H = 1$ and $N_L = 1$) with $m = 5$, $p = 10$, and $(n_w - n_r/(n_w + n_r) = 0.61$ gives +1.8 per cent and −2.0 per cent for the supply voltage tolerance, which is in good agreement with the graphical solution.

An examination of the case of three tubes in series where $N_H = 2$, $N_L = 1$, $m = 5$, $p = 10$, and $(n_w - n_r)/(n_w + n_r) = 0.61$ gives the tolerance on the supply voltage as −1 per cent and −4.7 per cent. In other words, the supply voltage can never be permitted to rise to normal without exceeding the maximum voltage rating on the low filament-current tube.

Conclusions

The curves given in Fig. 1 enable the transposition of heater or filament operating conditions within the usual desired engineering accuracy for electron-tube applications. These curves should not be used when an accuracy within ±3.5 per cent for filament current is desired with a ±25 per cent change in filament voltage. The percentage of error in all equations converges to zero as the ratio of E_x/E_0 approaches unity, or the smaller the percentage change of voltage in the transposition, the smaller the degree of error. A limited number of types of filaments and heaters have been examined and found to give good agreement with the curves of Fig. 1. The accuracy of the volt-ampere characteristic can be established by experimentally determining $(n_w - n_r)/(n_w + n_r)$ for the particular application. This determination may be made from the relation

$$\frac{n_w - n_r}{n_w + n_r} = \frac{\log (E_1/E_2)}{\log (I_1/I_2)}$$

where I_1, E_1 and I_2, E_2 are currents and voltages at known operating points.

Once the value of $(n_w - n_r)/(n_w + n_r)$ has been established over the probable application range of voltage, the operating conditions of a group of filaments in series may then be predicted for various supply voltages. This permits the supply voltage tolerance to be established so that the applied filament voltage tolerance may not be exceeded.

The solutions given here represent the steady-state conditions and do not indicate what may happen during the initial application of voltage or for short-time voltage transients.

References

(1) Jones and Langmuir, The Characteristics of Tungsten Filaments as Functions of Temperature, *Gen. Elec. Rev.*, June, July, and August, 1927.

(2) "Temperature—Its Measurement and Control in Science and Industry," p. 1186, American Physical Society, Reinhold Publishing Corporation, New York.

(3) Worthing, A. G., Physical Properties of Well seasoned Molybdenum and Tantalum as a Function of Temperature, *Phys. Rev.*, 28, No. 1.

(4) Mellor, Joseph, W., "Comprehensive Treatise on Inorganic and Theoretical Chemistry," Longmans, Green and Company, New York.

(5) Forsythe, W. E., "Measurements of Radiant Energy," p. 32, McGraw-Hill Book Company, Inc., New York.